sección especial en idioma inglés

on the fundamentals of economic evaluation

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ABSTRACT

This article develops a rationale for economic evaluation based on the premise that one's economic objective is to maximize his own wealth. This leads to a reconciliation between the views of those supporting the internal rate of return method and those supporting present worth methods of analysis. It also leads to a reconciliation between the major schools of thought regarding appropriate methodology for dealing with problems in which multiple rates of return are possible.

INTRODUCTION

The literature of Engineering Economics abounds in articles setting forth the virtues of the methodology alternatively as the internal rate of return, the discounted cash flow, or the profitability index method. It likewise abounds in articles lauding the present worth method, or that of its close relative, the average annual cost approach. The chief difference between these two approaches, both of which are based on compound interest theory is that the first seeks to solve for an "internal" rate of return, generally using a format that reduces all cash flows to their equivalent present value at varying rates of return, whereas the second assumes an interest rate, and then compares the present worths or the annual costs of the various alternatives under consideration. Among the terms used to describe the interest rate used are "minimum attractive rate of return", "the cost of capital," or simply "the rate of return."

Still more articles are written in which the author presumes one or the other of these two approaches to be more appropriate, perhaps justifying his choice by means of a footnote which refers to one of the articles written to support that particular view.

A few articles have been written attempting to resolve the controversy generated by proponents of these two methods, but we have yet to find one that seems to go to the heart of the matter; that is, to start by examining the true objectives of a decision-maker.

We will be so bold as to state what we feel that objective to be, go on to develop a methodology directed at satisfying that objective and describe a useful tool displaying all the information required (in a deterministic world) for making decisions in accordance with that objective. We shall touch briefly on the extension of the tool to probabilistic models, but leave a full exploration of this fascinating subject for subsequent articles. We shall attempt to make our assumptions and line of reasoning quite explicit in order to facilitate full discussion for our ideas.

WEALTH

Before one chooses among alternative courses of action, or selects a methodology for making such choices. he must properly start by examining his ojectives. In the real world of decision-making, these often turn out to be multi-dimensional; but in the relatively narrow world of economics, we will take as an axiom the economic objective of all expenditures is to maximixe wealth. This we hold to be true for both inidvidual and corporate persons. (To simplify matters, we shall, in the remainder of this article, speak of both as "a person.")

Since one's present wealth is a fixed amount (generally taken to be too small), should perhaps be more explicit and state that we assume that it is the economic objective of every person to maximize his future wealth. But here we run into a difficulty, for the future runs from here to infinity. Do not despair; we shall attempt to give an operationally useful definition of future as we develop our ideas.

How can a person evaluate his future wealth? His present wealth can grow in many different ways; for example:

A He can invest in a savings account and reinvest all accrued interest. Then his wealth will grow at, say, 4½ percent per year.

B) He can buy bonds, yielding perhaps 8 percent, and with the interest payments buy more bonds of the same type. His growth rate in this case is 8 percent yearly.

C) He can enter the stock market. In this case his wealth may grow in an erratic manner, perhaps, growing faster than it would in the first two cases in good years and less rapidly —or even negatively— in bad times. His wealth at any time will be a function of the stock market behavior to that point and his method of reinvestment of dividends and proceeds from stocks sales.

In all cases, a key factor in his future wealth is seen to be the reinvestment rate.

MECHANICS OF EVALUATION

Given the future cash flow promised by each alternative project under consideration, we can make an evaluation of the future wealth that each promises, provided that we have some way to establish an appropriate reinvestment rate. We shall make use of a simple example to illustrate various ways of accomplishing this. This example is as follows:

We are asked to select one of two mutually exclusive projects, which, for simplicity, we shall call project A and project B.

Project A requires an investment of $30,740 and promises a cash inflow of $11,320 at the end of each year for ten years. Project B costs $10,000 and will produce a $4,610 cash inflow at the end of each year for ten years. Calculations by the usual methods show the internal rate of return for project A to be 35 percent; that for B to be 45 percent.

In the following discussion we shall assume that, in the rich language of the decision analyst, A, B, and "Do Neither" are mutually exclusive and collectively exhaustive present alternatives and call their corresponding choices III, II and I respectively. We shall also assume that the decision-maker has at least $30,740 at his disposal.

A reasonably complete description of the three choices available follows:

I) Invest in neither A nor B. this case we assume he would not put his present welath under a mattress; instead, he would let his entire wealth grow elsewhere at a rate we shall call X percent per year. Thus, it is seen that "do neither A nor B" is not properly called a "DO NOTHING" alternative.

II) Invest $10,000 in project B and let the th reemainder grow at interest rate X elsewhere. (We here assume that an investment in B will not affect the investment opportunities available elsewhere.) We recognize that for very small amunts our assumption that any amount can be invested at rate X may be unrealistic, but suggest that, for decisions addressed by engineering economists, even small amounts are cumulated to the degree that our assumption becomes reasonably realistic. (For a slightly more complex assumption becomes reasonably realistic. (For a a slightly more complex assumption, see Thuesen or a summary in reference 13.)

III) Invest $30,740 in project A and allow the remainder to grow at rate X. We here note that any amount over $30,740 is common to all three alternatives and is therefore irrevelant to our choice among them. We will therefore ignore such sums in our remaining discussion.

In everything said up to now, we assume that X is the prediction of the rate at which we can employ our resources over the next ten years in projects other than A or B. For the moment we assume that the predicted rate is not a function of time, and we continue to live in a deterministic world. A similar line of reasoning to that developed thus can also lead (conceptually, at least, to a solution of the capital budgeting problem, that is, one which all possible combinations of alternatives are possible, subject to certain constraints.

To meet our objective we should choose that alternative which leads to the maximum wealth at the end of 10 years —the earliest possible common time horizon.

We here digress briefly to return to the question, earlier implied, "When, in the future, do we wish to maximize our wealth?"

We are now ready to offer an operational answer to that question. In the decision, one should attempt to maximize the wealth at the end of the shortest common time horizon for all alternatives under consideration, on the basis that anything that happens after that time is common to all alternatives being considered and therefore irrevelant to the present decision.

Returning to our problem, we can calculate the wealth that each alternative would lead us to as a function of the reinvestment, or growth, rate X and display this information as shown in Figure 1.

This graph offers the following information:

A) If we foresee a reinvestment rate from zero to 30 percent per year the optimal decisión is to invest in A.

B) If we foresee a reinvestment rate which is between 30 and 45 percent the best thing to do is to accept B.

C) For any reinvestment (or growth) rate above 45 percent, one should reject both A and B in order to maximize future wealth.

As has been noted in many places (see, for example references 1, 2, and 2), a decision to choose the project yielding the largest internal rate of return can lead to a non-optimal decision. It is generally recommended that this difficulty be avoided by calculating another rate of return, often called "the rate of return on extra investment," at which a pair of alternatives are equally attractive. In our example, if B is found to be attractive, the decision as to whether or not to take A instead would be based on the calculation of the rate of return on the extra investment in A (as compared to B.) In numbers, this extra investment is $20,740; the additional cash inflow of $6,710 per year for 10 years represents a rate of return on extra investment of 30 percent. (Note that this value is obtained directly from Figure 1.) At the risk of being repetitious, we again emphasize that the calculation so obtained is irrevelant if B is unattractive.

We have deliberately chosen a simple example, involving only two alternatives and in which the time horizons for the alternatives are the same and in which no negative cash flows follow the first positive cash flow. Before going on to more complex cases, perhaps we should pause to propose to us now seems a meaningful definition of that which is usually called "the internal rate of return." We would define this term to mean that growth (or reinvestment) rate for which one's wealth, at the earliest common time horizon, for two alternatives is the same. (Remember, if there are more than two alternatives, they must be compared in appropriate pairs if any rate of return approach is to be meaningful!)

Is it not obvious that a graph of future wealth vs. growth, or reinvestment, rate can be plotted for all mutually exclusive projects, or combinations of projects, available in order to select among any number of projects? All such graphs will contain the alternative of rejecting all projects or combinations thereof (this might be called the null alternative); and in such a graph the optimal policy will be a piecewise curvilinear function of the reinvestment, or growth rate.

FUTURE WORTH

Let us define future worth as the potential increment in wealth that a project possesses, as compared to the null alternative, at the end of the contemplated horizon. This can be plotted as a function of the growth or reinvestment, rate and such a plot is shown in Figure 2 for projects A and B.

As would be expected the plotted lines corresponding to the future worths cross each other where the wealths obtained from choice II and III cross in Figure 1 and they cross horizontal axis where the respective choices cross choice 1 in this same figure. We obtain the same information from this graph as from Figure 1, namely:

A) Between 0 and 30 percent, A is the best alternative.

B) Between 30 and 45 percent, we should choose B.

C) Above 45 percent, both should be rejected.

D) The internal rate of return of project A is 35 percent.

E) The internal rate of return of project B is 45 percent.

F) The internal rate of return on extra investment in A is 30 percent.

FIG. 1.-WEALTH VS. GROWTH RATE.

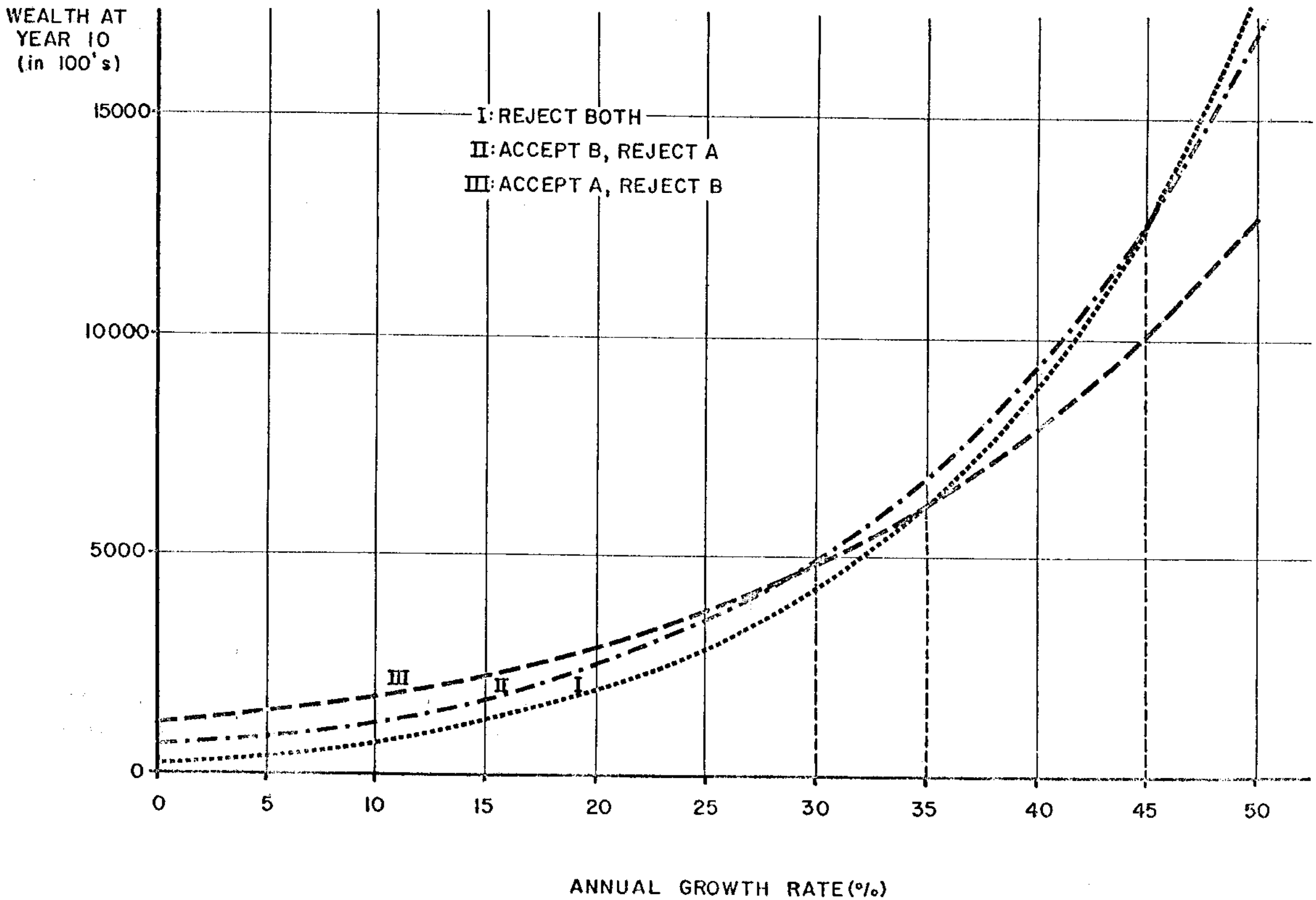
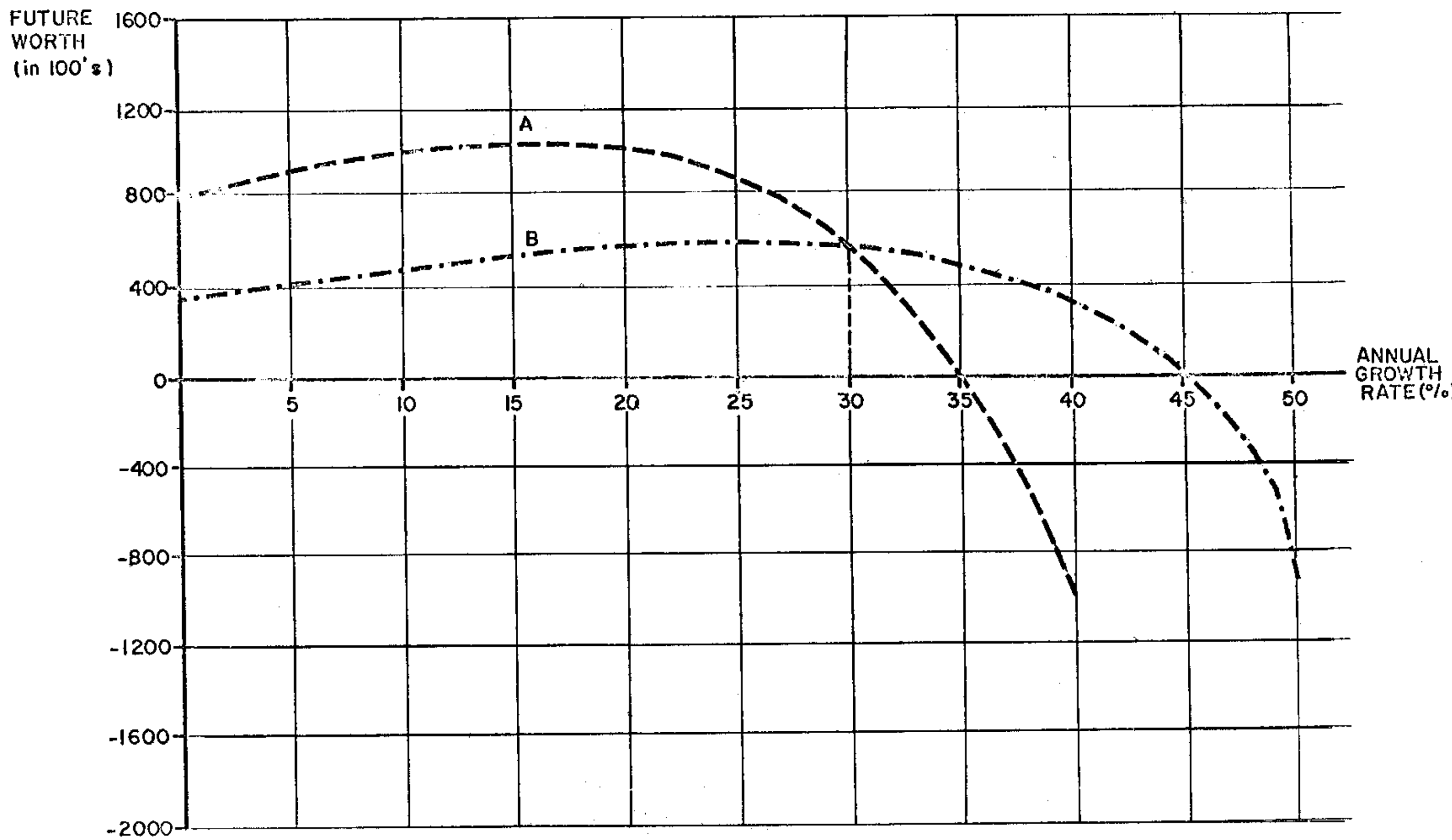


FIG. 2.-FUTURE WORTH VS. GROWTH RATE.



PRESENT WORTH

It is simple to show that, if discounting is done at the reinvestment rate, the present and future values of any alternative (including those of the differences between any pair of alternatives), differ only by a scale factor —that scale factor being the present worth factor, single payment (or the P/F factor)— for that reinvestment rate and the common planning horizon.

Thus the future worth of a series of cash flows in given by

$$FW = \sum_{n=0}^{N} V_n \, (1+i)^{N-n}$$

where Vn is the cash flow at the end of the nth period and i is the reinvestment rate.

Bringing this back to the presnt, using

Bringing this back to the present, using $P = F (1+i)^{-N}$:

$$PW = (1+i)^{-N} \sum_{n=0}^{N} V_n \, (1+i)^{N-n}$$

$$= \sum_{n=0}^{N} V_n \, (1+i)^{-n}$$

This, of course, is precisely the formula for the present worth of a series of cash flows at interest rate i.

These relationships tell us that any inferences drawn from a future worth comparison can be obtained equally well from a present worth comparison —a conclusion that is hardly surprising. Our reasoning, through, says that the logical comparison is on a Future Worth basis; the familiar Present Worth basis offer a convenient and familiar algorithm that leads to the same conclusion!

Following this line of reasoning, Figure 3 shows the present worth of each project as a function of the reinvestment rate (often called, in this context, the discouting rate).

As did Figure 2, it tells us that:

A) A is the best between 0 and 30 percent.

B) B is the best between 30 and 45 percent.

C) Above 45 percent, none is good.

D) Rate of return of A: 35 percent.

E) Rate of return of B: 45 percent.

F) Rate of return of A over B: 30 percent.

Both the Future Worth and Present Worth graphs offer more information than that, however. They also tell us how much we must forego if, for noneconomic reasons, we wish to take a non-optimal project. They provide us with information about our bargaining position in mergers, acquisitions and all sorts of financial dealings and they yield information regarding the sensitivity of the outcomes to errors in prediction in a world recognized as uncertain.

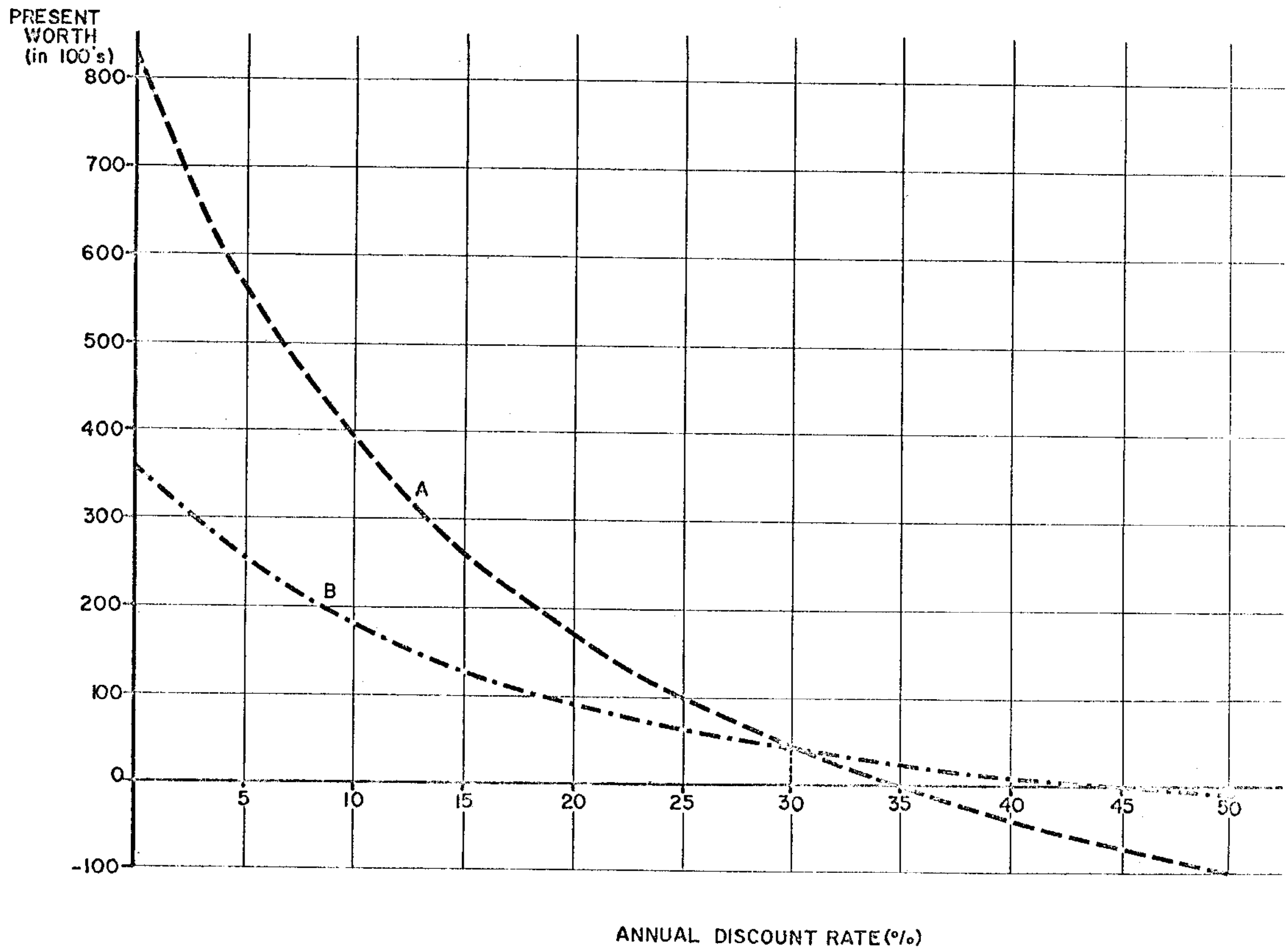
REVERSALS OF SIGN IN CASH FLOW SERIES

There are occasional instances in which proposals involve more than one reversal in sign of the prospective cash flows (see references 3 and Appendix B of reference 6). In the literature there is much discussion of the difficulty posed by the fact that the usual discounted cash flow approach can lead to no, one, or more than one, real positive roots. This should not be surprising, since the present worth equation is

$$PW = \sum_{n=0}^{N} V_n \, (1+i)^{-n}$$

This is a polynomial of the Nth degree, and such a polymonial has N roots, all of them complex numbers: $a + j\,b$, where j stands for the square root of -1. The only roots that are meaningful from the economic point of view are those in which a is positive or negative (there can be a negative growth rate in the future which means

FIG. 3.- PRESENT WORTH VS. DISCOUNT RATE.



that our wealth will decrease) and b is zero. Descartes' rule of signs helps us predict the maximum number of real roots —but that need not concern us here. As an example, let us consider the following problem:

A contract is available which pays $8,000 "front money" upon the signing of a contract, after which expenses of $22,000 are incurred at the end of the first year, and an income of $15,000 is received at the end of the second year. (As elsewhere in this article, we use the end of year convention for simplicity of discussion.)

This leads to the cash flow diagram of

TIME	CASH FLOW
0	+ 8,000
End of Year 1	− 22,000
Endr of Year 2	+ 15,000

The equation for the present worth of this project is

$$PW = 8{,}000 - 22{,}000\ (1+i)^{-1} + 15{,}000\ (1+i)^{-2}.$$

If we set this equal to zero, there are two values of i which will satisfy the equation:

$i_1 = .25$ (25%)

$i_2 = .50$ (50%)

How should one interpret such strange results in an economic sense?

To be consistent, we must apply exactly the same criterion as in other problems; that is, accept the alternative (take contract or refuse contract) leading to the greater future wealth. (As always, in making these statements, we assume there to be no other specific alternative now available or crearly foreseen!) We will analyze thoroughly the first case for illustration.

To make "accepting the contract" a viable alternative, one must have $22,000 available at the end of the first year. This means that, if the reinvestment rate is X, he must have $22{,}000\ (1+X)^{-1} - \$8{,}000$ available at time zero. If one invests this sum, and it grows to $22,000 at the end of year one and this is paid out to cover expenses, then one receives $15,000 at the end of the second year. One's second alternative is to reject the contract, investing $22{,}000\ (1+X)^{-1} - 8{,}000$ elsewhere, letting it grow at x percent per year. These alternatives lead to the following wealths at the end of the second year

Accept: $W_1 = 15{,}000$

Reject: $W_2 = [22{,}000\ (1+X)^{-1} - 8{,}000]\ (1+X)^2$

which, upon simplification becomes
$W_2 = 22{,}000\ (1+X) - 8{,}000\ (1+X)^2$

The difference between W_1 and W_2, which we have defined as future worth, is

$$FW = 15{,}000 - 22{,}000\ (1+X) + 8{,}000\ (1+X)^2.$$

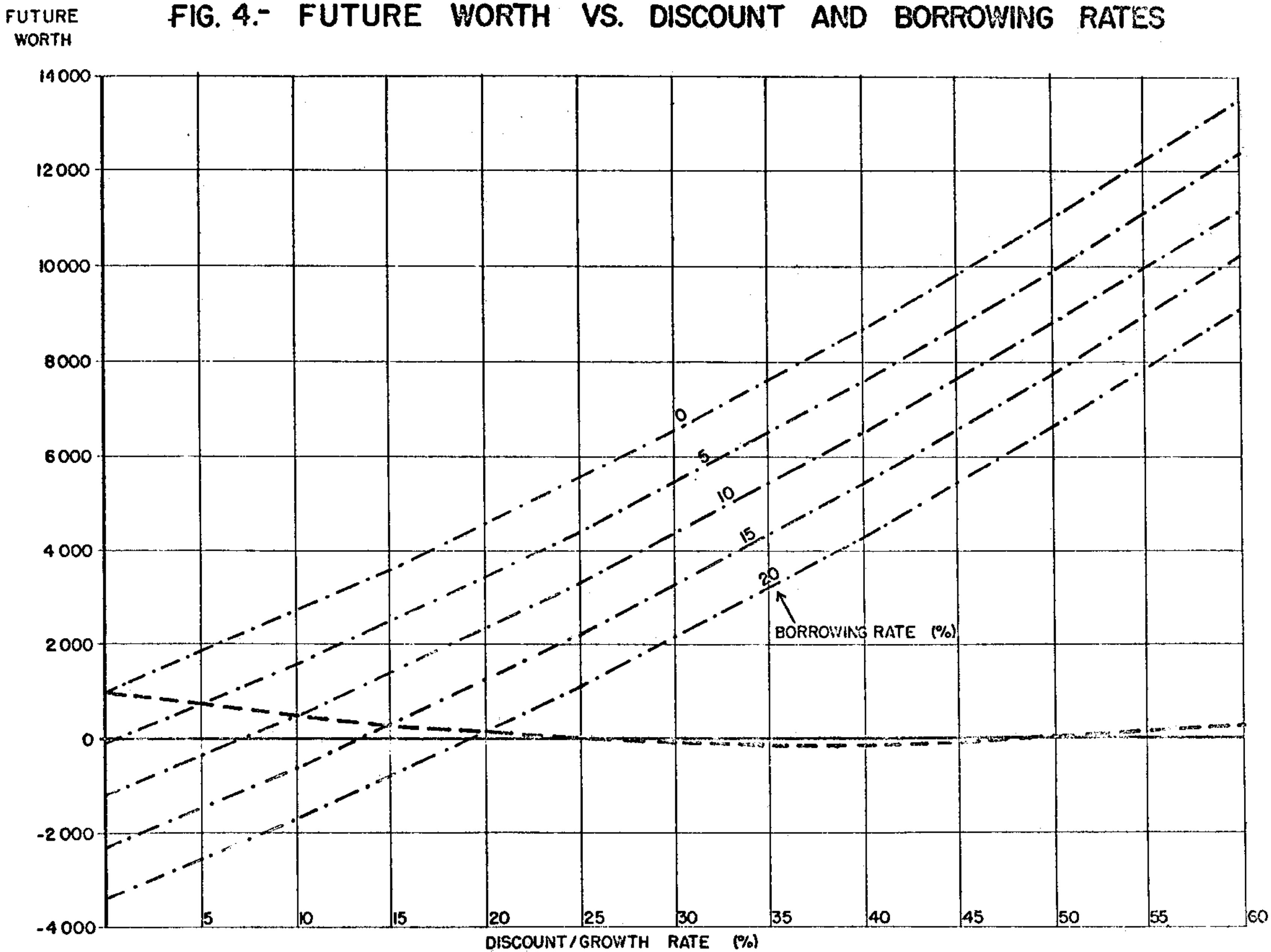
This is plotted as a function of X in Figure 4, as a dashed line, which shows that for reinvestment rates up to 25 percent and over 50 percent the contract is attractive; for reinvestment rates etween 25 and 50 percent it is not.

By now we know that a Present Worth vs. Reinvestment Rate graph would offer precisely the same information, but that the vertical axis on the graph would change by a factor of $(1+X)^{-2}$.

Bill Morris has suggested an interesting variant of this problem in which the initial payment is +$10,000, the first year cost −$25,000. The two solution rates in this case are $0.5 + j0.5$ and $0.5 - j0.5$— both complex numbers!

A Future, or a Present Worth, graph will inmediately show that it is an attractive alternative for any positive reinvestment rate! (A Present Worth graph is shown as the dashed line in Figure 5.)

FIG. 4.- FUTURE WORTH VS. DISCOUNT AND BORROWING RATES



Many suggestions have been made since Solomon [3] first raised the issue of multiple rates of return; for example, Appendix B of Grant Ireson [6] on page 552 offers the following statement:

> The key to an evaluation of such proposals lies in the use of an auxiliary interest rate. We shall also see that an important aspect of the matter is the sensitivity of the conclusions of an evaluation to moderate changes in the auxiliary interest rate selected.

The authors then go on to explain how this auxiliary interest rate should be used, in a way which seems to contradict what they say in Chapter 18: that decisions regarding the source of capital funds and their investment should be made separately.

In this appendix, the "auxiliary interest rate" (Ruel's [7] name for it is the "crutch rate") is used throughout in the same manner as our "growth rate" for all negative cash flows while using a different "discount/growth rate" for all positive cash flows.

Given our assumptions, it seems quite clear that the "discount" and "reinvestment" rates should be one and the same, since the question being addressed is, "For what range of reinvestment rate is the proposal attractive?" or, alternately, "At what reinvestment is it just break even?"

Thus, it is seen that adopting the criterion of maximizing future wealth brings about a reconciliation of the argument between two camps regarding the proper way to analyze cash flows containing multiple sign changes. One camp, exemplified by Thuesen, Fabrycky, & Thuesen, holds that a Present Worth (or by implication, a future Worth) graph will show those ranges of growth rate for which a project is attractive and a second, discussed above, which holds that an auxiliary, or crutch rate, should be used to give a unique solution. (We note in passing that one author has embraced both approaches —the former in his book and the latter in a subsequent article in this Journal.)

If we look upon a discounting rate as identical to the reinvestment rate, is it not clear that a project should be accepted if its Present (or Future) Worth at the appropriate reinvestment rate is positive? and that is should be rejected if its Present Worth (or Future Worth) is negative?

Is it not further clear, that one who chooses to solve for an unknown rate should seek for a reinvestment rate, and that therefore his "crutch rate" should, in fact, be the solution rate?

In the example we have been examining, either a Future Worth or a Present Worth vs. Growth Rate graph will show the range of reinvestment or growth) rates for which the proposal is acceptable.

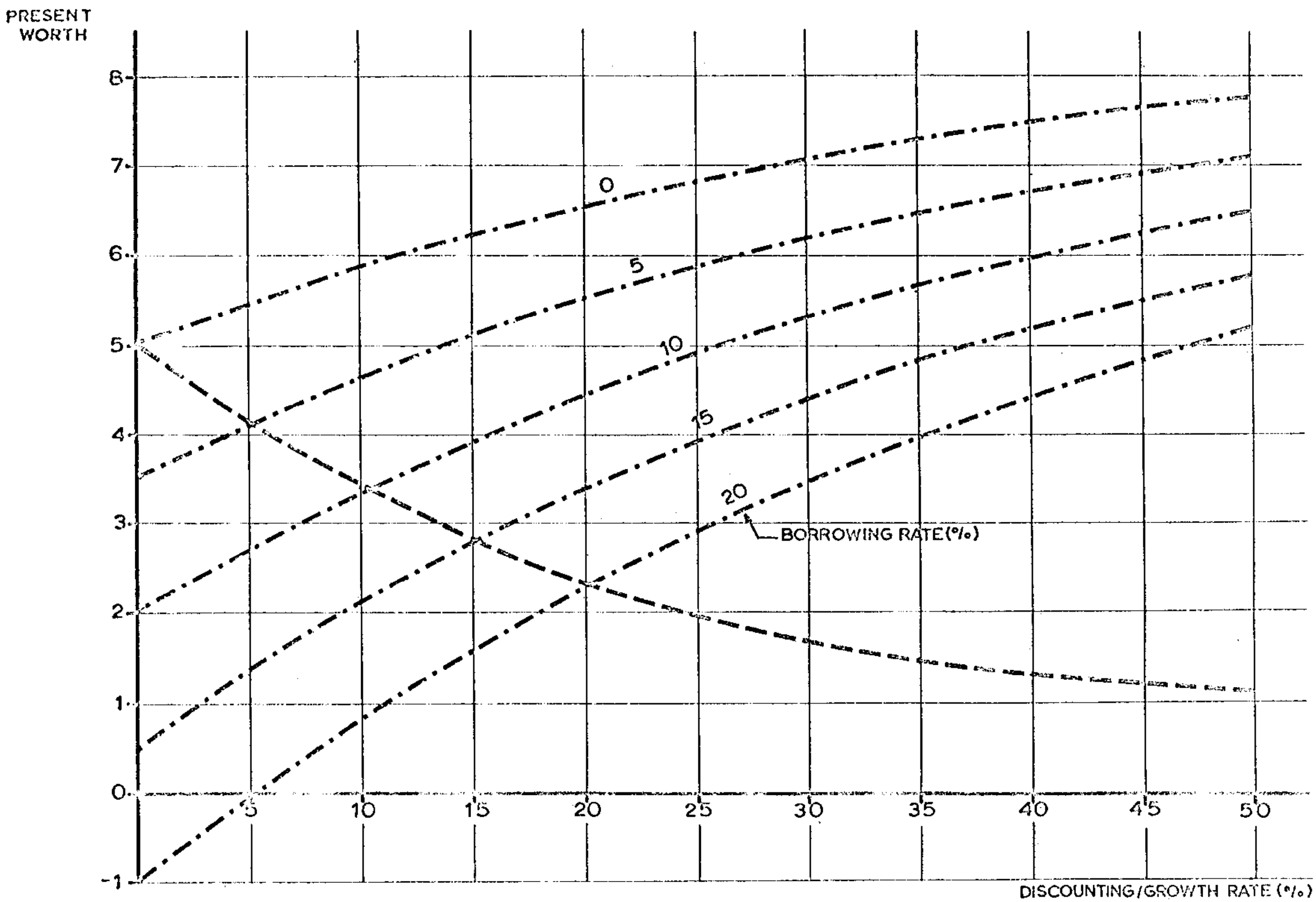
Of course, if the decision-maker does not have wealth necessary to obtain a wealth of \$22,000 by the end of the first period, but wishes to borrow the money at that time, or if the reinvestment rate is different for each period, a different model must be used.

For example, to solve the first case, we must first more explicit about the actual cash flows. Let us assume the decision-maker will borrow the entire \$22,000 at the end of the first year and that he wishes to repay the loan at the end of the second year. If we designate his borrowing rates as b and the growth (or reinvestment) rate as i, the equation yielding his wealth at the end of year 2 is given by

$$FW = 8{,}000\,(1+i)^2 - 22{,}000\,(1+b) + 15{,}000.$$

This can be portrayed as a three dimensional graph, or somewhat more conveniently, for selected values of the borrowing rate, as in Figure 4 by the dash-point lines. These show the Future Worth of the project at various borrowing rates and reinvestment rates; if the Future Worth for a given combination of the two is positive, the project should be undertaken. For example, if the borrowing rate is 20 percent, the graph tells us that unless the reinvestment rate is greater than about 18 percent, the project should be rejected. The dash- point lines on Figure 5 offer a solution to a similar interpre-

FIG. 5.-PRESENT WORTH VS. DISCOUNT AND BORROWING RATES.



tation of the Morris problem. Of course, a present worth graph could be used in the same manner.

PREDICTING THE GROWTH RATE

The determination of a reinvestment (discounting) rate is, in all sense, the most challenging aspect in the economic evaluation since there is no fail proof method of "knowing what will happen."

The determination of the "future price of money" is a problem in forecasting, one of probing into the future. and given that uncertainty exists in everything regarding the future, the analyst may have to resort to sophisticated methods of helping the decision-maker by taking into account such factors as risk preferences. John R. Canada [4] writes:

> A very pertinent question to the economic analyst is: Just what reinvestment rate is appropriate? The answer which is obvious in words but which involves the difficulties of forecasting in order to translate into specific figures is: The rate at which the recovered capital can be or will be reinvested.

and he attempts to determine this figure through

> . . .a weighted average of expected future opportunities which would be accepted over the period in which the capital recovered from the project being currently studied will be available for reinvestment.

We can see two difficulties in this: 1) you need a crystal ball to know the "future opportunities" and 2) even if you know them, the use of a single weighted average for all time periods may be too gross a tool.

Many writers advocate the use of the "cost of capital" as the discouting (reinvestment) rate, the method of determination of which has been the subject for many conflicting articles. To look upon the past cost of capital, however calculated, as an estimator of future reinvestment rates seems to us an exercise in futility.

Hopefully, the mean growth rate in the time span from now to the planning horizon will surpass the mean cost of capital in the same period ut there is no definite relationship, at least deterministically, between the two.

Others advocate the use of a rate that goes by a very colorful name: "minimum attractive rate of return," and then go through an effort to show how this rate can be determined by internal capital rationing. Again we must question the validity on such a figure as a forecaster of how our wealth is going to grow.

If we agree on the fact that the growth rate for the future is uncertain (very few things are not), then we must also agree that we must either predict a single rate for all periods or resort to more sophisticated techniques of analyzing decisions: techniques which take into account such factors as the uncertainty existing in the future and the risk preference of the decision-makers. One technique for doing that is described by Howard [5], who states ". . . selecting the appropriate interest rate is not easy; it invoives the nature of the interaction between the organization and its financial environment." Another phrase which is very indicative of the kind of problem we are facing is the following: "The interest rate is not only an expression of the force of nature; it also depends on the wisdom of men" [8]; from both expressions above we can see that the growth rate we have been talking about throughout this article is a "state variable" (Decision Analysis terminology): one which is not under control of the decision-maker; and until we know 'how it depends on the wisdom of men" and how to measure this wisdom, we must keep on encoding it in our analysis strictly as a force of nature.

DIFFERING LIVES

The problem posed when alternatives have differing lives is not a trivial one.

In much of the literature (and this is particulary true of articles advocating a return on investment approach) the problem is "solved" by considering only examples in which the lives are the same; where the problem is addressed, it is often suggested that it be solved by using a uniform annual cash flow approach. This is supportable only on the assumption that the need is for some common multiple of the lives of the alternatives under consideration (including infinity), that technological changes will not take place, and that cost history will repeat except for changes completely responsibe to any inflation rate.

In many cases these assumtions are reasonably realistic; when they are not, it is obvious that a "future history" projection must be made to some common termination date. For example, in deciding to invest in two proposals of differing lives, both of which are determined by technological factors, it is clear that one should attempt to calculate one's wealth as of the end of the life of the longer lived alternative. Lacking other information, one would project, as usual, reinvestment at the growth rate until that time.

Since uniform annual cash flows can be obtained by multiplying present worths by (A/P, i, n) or future worths by (A/F, i, n) (we here use standard engineering economy symbols) it is clear that any information offered by a Future Worth or Present Worth vs. Growth Rate graph can be obtained equally well by a Uniform Equivalent Annual Cash Flow vs. Growth Rate graph. (To make the title less of a mouthful, we shall refer to this as a Uniform Flow vs. Growth Rate graph.)

Capitalized Flows is nothing but a Present Worth for perpetual operation, and we already saw that decisions under Present Worth for perpetual operation, and we already saw decisions under Present Worth or wealth maximization criteria are identical, although speaking of wealth at the end of an infinite number of years is not very meaningful.

In any case a Uniform or Capitalized Flows vs. Reinvestment Rate graph will provide us with the same type of information yielded by the graphs discussed thus far:

A) Ranges of growth rates for which each alternative is superior to the rest.

B) Internal rates of return of the proposals, by themselves and taken as pairs, or rates of return of the extra investments, if economically meaningful.

C) Amounts you are required to sacrifice due to non-economic reasons for implementing other than the best alternative.

D) Ranges of monetary values for financial transactions.

CONCLUSIONS

We believe that most of the controversy regarding criteria for economic evaluation can be traced to the fact that we have been overemphasizing the search of one single number and/or method that will lead to a decision. This has made us forget our objective (or fail to see the forest for the trees); we have been striving for the means while losing sight of the end.

1. If one's objective is to maximize wealth, either a Future Worth, Present Worth, or Capital Flow vs. the Growth, or Reinvestment Rate graph has been shown to be a useful tool for arriving at an optimal decision. Such graphs also display the results sought by the return on investment method and are useful in making sensitivity studies.

2. The plotting of such graphs will not demand more effort than that required for other methods of solving for unknown interest rates. Indeed, computer programs can be simplified by eliminating trial and error or looping subroutines —and it is no great task to have the computer print the graphs directly— also it will look much nicer.

3. The use of these graphs tends to eliminate the difference between 'present worth cultists" and those seeking a return-on-investment type solution.

4. We have shown the use of "a crutch rate' in solving problms leading to multiple rates of return to be unnecessary unless borrowing is indispensable and offered a variant of the standard graph to take care of this case.

5. We have pointed out that the growth

rate is, in the terminology of decision analysis, a state variable and have suggested that if the simple projection of a time-invariant rate is not reasonable, advanced techniques for prediction of the rate as a function of time and correspondingly more complex models will be required.

6. Although we have shown that Future Worth, Present Worth or Capital Flow vs. Growth Rate graphs all lead to identical conclusions, we feel that the Future Worth graph focuses more directly on a decision-marker's time objective to maximize future wealth. On the other hand, the use of present worth concepts is more common and so others may prefer to use the present worth form since it "fits like an old shoe."

7. While we do not claim to have discovered such graphs, we do wish to claim the right of naming them. We propose that they be termed G & I graphs, in honor of the authors of the most widely used book on engineering economy, both of whom are fine teachers and true gentlemen.

8. If others refuse to accept an axiom that economic man strives to optimize his own wealth —or deny the usefulness of the economic man concept, it is incumbent upon them to propose alternative axioms and to develop methods of assessment that speak to such objectives. An example of this is a recent contribution by Pollard [11] who examines a "Consumption Preference" approach introduced by Fisher [12] and developed by Hirshleifer [13].

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